Code No. E-10315

FACULTY OF SCIENCE

B.Sc. (CBCS) IV- Semester (Regular & Backlog) Examination, June / July 2023

Subject: Mathematics

Paper- IV: Algebra

Time: 3 Hours **Max. Marks: 80**

PART – A

(8 x 4 = 32 Marks)

Note: Answer any eight questions.

1. In a group G, show that inverse of an element is always unique.
2. Show that $U(14) = \langle 3 \rangle = \langle 5 \rangle$.
3. Find the inverse of $A = \begin{bmatrix} 4 & 3 \\ 6 & 5 \end{bmatrix}$ in the group $GL(2, \mathbb{Z}_7)$.
4. In S_5, if $\beta = (1\ 3\ 5\ 4\ 2)$ then find β^2.
5. Show that any finite cyclic group of order n is isomorphic to $\mathbb{Z}_n$.
6. Let H be a sub group of a group G and $a, b \in G$. Show that $aH = bH \Leftrightarrow a^{-1} b \in H$.
7. Show that every subgroup of an Abelian group is a normal subgroup.
8. Show that the mapping $\phi: \mathbb{R}^* \to \mathbb{R}^*$ defined by $\phi(x) = |x|$ is a homomorphism. (Here $\mathbb{R}^*$ is the group of non-zero real numbers under multiplication.
9. Let R be a ring and $a \in R$. Show that the set $S = \{x \in R | ax = 0\}$ is a subring of R.
10. If A and B are any two ideals of a ring R, then show that $A \cap B$ is also an ideal of R.
11. Find all idempotent elements in the ring $\mathbb{Z}_{20}$.
12. Find all solutions of $x^2 - 6x + 8 = 0$ in the ring $\mathbb{Z}_4$.

PART – B

(4 x 12 = 48 Marks)

Note: Answer all the questions.

13. (a) Let G be a group and H be a non empty subset of G. The show that H is a subgroup of G if and only $ab^{-1} \in H$ whenever $a, b \in H$.

(OR)

(b) (i) Show that every subgroup of a cyclic group is cyclic
(ii) Let G be a group and $a \in G$. Then show that $\langle a \rangle = \langle a^{-1} \rangle$

14. (a) Show that the disjoint cycles commute.

(OR)

(b) Let $\phi: G \to \bar{G}$ be an isomorphism. Then show that
(i) $|a| = |\phi(a)|$ for all $a \in G$.
(ii) G is Abelian if and only if $\bar{G}$ is Abelian.

15. (a) Determine all group homomorphisms from $\mathbb{Z}_{12}$ *to* $\mathbb{Z}_{30}$.

(OR)

(b) State and prove the first isomorphism theorem on groups.

16. (a) Show that the ideal $\langle x^2 + 1 \rangle$ is a maximal ideal in the ring $\mathbb{R}[x]$.

(OR)

(b) If A, B are any two ideals in a ring R, then show that $A + B = \{a + b | a \in A, b \in B\}$ is an ideal of R.

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Code No. D-6415

FACULTY OF SCIENCE

B.A / B. Sc. (CBCS) IV - Semester (Regular & Backlog) Examination, June / July 2022

Subject: Mathematics
Paper - IV (Algebra)

Time: 3 Hours | Max. Marks: 80

PART – A

Note: Answer any eight questions. (8 x 4 = 32 Marks)

1. Show that the set of all 2x2 matrices with determinant 1 with entries from Q(rationals) is a non-abelian group under matrix multiplication.

2. Find the order of all elements of Z_{10} under addition modulo 10.

3. Show that U(8) is not a cyclic group.

4. Suppose $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 3 & 5 & 4 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 1 & 2 & 3 \end{bmatrix}$ are two permutations. Then express $\alpha\beta$ as disjoint cyclic form.

5. Define a coset and write any four of its properties.

6. Suppose G is a finite group and let a G. Then prove that $a^{|G|} = e$.

7. Prove that the centre Z(G) of group is always normal.

8. Define ring and give an example of a commutative ring without unity.

9. Show that the ring of Gaussian integers $Z(i) =\{a+bi / a,b \in Z)$ is an integral domain.

10. Define ideal and mention the ideals of R.

11. Show that the ideal $\langle x^2+1 \rangle$ is not prime in $Z_2[x]$.

12. Determine all ring homorphisms for Z_{12} to Z_{30}.

PART – B

Note: Answer all the questions. **(4 x 12 = 48 Marks)**

13. (a) Suppose H is a non-empty finite subset of a group G, If H is closed under the operation of G, then prove that H is a subgroup of G.

(OR)

(b) State and prove fundamental theorem of cyclic groups.

14. (a) Prove that every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.

(OR)

(b) State and prove Cayley's theorem.

15. (a) Suppose G is a group and let H be a normal subgroup of G. Then prove that the set $\frac{G}{H} = \{aH/a \in G\}$ is a group under the operation (aH)(bH) = abH.

(OR)

(b) Define characteristic of a ring. Prove that the characteristic of an integral domain is zero or prime.

16. (a) Suppose R is a ring and let A be a subring of R. Then prove that the set of cosets $\{r+A/r \in R\}$ is a ring under the operations (s+A)+(t+A) = s+t+A and (s+A)(t+A) = st+A if and only if A is an ideal of R.

(OR)

(b) Suppose R is a commutative ring with unity and let A be an ideal of R. Then prove that $\frac{R}{A}$ is field and only if A is maximal.

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Sem-4
(maths)

Code No. 18123/N

FACULTY OF SCIENCE

B.A./B.Sc. I Semester (CBCS) Examination, September / October 2021

Subject: Mathematics

Paper – IV : Algebra

Time: 2 Hours | Max. Marks: 80

PART – A

Note: Answer any five questions. (5 x 4 = 20 Marks)

1 Show that the set {0,1,2,3} is not a group under multiplication modulo 4.
2 Find the order of all elements in U(15) under multiplication modulo 15.
3 Find the number of generators of the set Z_8.
4 Show that the symmetric group S_3 is non-abelian.
5 Suppose $G=S_3$ and H={(1), (13)} then find the right cosets of H in G.
6 Prove that a group of prime order is cyclic.
7 Prove that every subgroup of an abelian group is normal.
8 Define ring and give an example of a non-commutative ring with unity.
9 Show that $Z(\sqrt{2})=\{a+b\sqrt{2}\,/\,a,b\in Z\}$ is an integral domain.
10 Define the terms ideal and principal ideal.
11 Show that the ideal $<x^2+1>$ is maximal in $R[x]$.
12 Define Ring Homomorphism and Ring Isomorphism.

PART – B

Note: Answer any three questions. (3 x 20 = 60 Marks)

13 Prove that center of a group is a subgroup of G.

14 Prove that every sub group of a cyclic group is cyclic.

15 State and prove Cayley's theorem.

16 State and prove Lagrange's theorem.

17 State and prove first Isomorphism theorem.

18 Prove that every finite integral domain is a field. Give an example of an infinite integral domain that is not a field.

19 Suppose R is a ring and let A be a subring of R. Then prove that the set of cosets {r+A/r∈R} is a ring under the operations (s+A)+(t+A)=s+t+A and (s+A) (t+A)=st+A if and only if A is an ideal of R.

20 Suppose R is a commutative ring with unity and let A be an ideal of R. Then prove that R/A is an integral domain if and only if A is prime ideal.

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Code No. 3121/E

FACULTY OF SCIENCE

B.Sc. IV-Semester (CBCS) Examination, May / June 2019

Subject : Mathematics

Paper – IV (DSC) : (Algebra)

Time : 3 Hours **Max. Marks: 80**

PART – A (5 x 4 = 20 Marks)
(Short Answer Type)

Note : Answer any FIVE of the following questions.

1 Prove that the set

$$GL(2, R) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Big/ a, b, c, d \in R,\ ad - bc \neq 0 \right\}$$

Is a non abelian group with respect to matrix multiplication.

2 Let G be a group and H be a nonempty subset of G. If $ab \in H\ \forall a, b \in H$ and $a^{-1} \in H\ \forall\ a \in H$ then prove that H is a subgroup of G.

3 State and prove Lagrange's theorem.

4 A subgroup H of G is normal in G if and only if $xHx^{-1} \subseteq H\ \forall x E H$.

5 Prove that the characteristic of an integral domain is either zero or prime.

6 Let R[x] denotes the set of all polynomials with real coefficients and let A denote the subset of all polynomials with constant term 0 then prove that A is an ideal of R [x] and A = < x >.

7 Let ϕ be a ring homomorphism from a ring R to a ring S then Ker $\varphi = \{r \in R / \varphi(r) = 0\}$ is an ideal of R.

8 If D is an integral domain then prove that D[x] is an integral domain.

PART – B (4 x 15 = 60 Marks)
(Essay Answer Type)

Note: Answer ALL from the questions.

9 (a) Every subgroup of a cyclic group is cyclic more over if $|\langle a\rangle| = n$ then the order of any subgroup of < a> is a divisor of n and for each positive divisor k of n, the group < a > has exactly one subgroup of order K namely < a > .

OR

(b) Define Alternating group of degree n . Also prove that A_n has order $\frac{n!}{2}$ if n > 1.

10 (a) Prove that the group of rotations of a cube is isomorphic to S_4.

OR

(b) Let G be a group and let Z(G) be the centre of G. If $\frac{G}{Z(G)}$ is cyclic then G is abelian.

11 (a) Prove that $Z_3[i] = \{a\ ib\ /\ a, b \in Z_3\}$ is a field of order 9.

OR

(b) Let R be a commutative ring with unity and let A be an ideal of R then $\frac{R}{A}$ is an integral domain if and only if A is prime ideal.

12 (a) If R is a ring with unity and the characteristics of R is n > 0 then prove that R contains a subring isomorphic to Z_n. If the characteristic of R is 0 then R contains a subring isomorphic to Z.

OR

(b) Let $S = \left\{ \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \ / \ a, b \in R \right\}$ then show that $\phi : \mathbb{C} \to S$ given by

ϕ (a + (b) = $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ is a ring isomorphism.

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